Datum/Zeit	Veranstaltungsort	Thema
Mo, 08.02.2010	Hörsaal Institut für Glaschemie	Albert Einstein and the Viscosity of
10.00-11.30	Fraunhoferstrasse 6	Macromolecules
Mo, 08.02.2010	Hörsaal Haus 1,IAAC,	Light Scattering and SEC-MALLs
12.15-13.45	August-Bebel-Str. 2	
Di, 09.02.2010	Institut für Materialwissenschaft und	Dynamic Light Scattering
12.15-13.45	Werkstofftechnologie, HS 124	
	Löbdergraben 32	
Mi, 10.02.2010	Hörsaal 3	Analytical Ultracentrifugation I
16.15 17.45	Carl Zeiss Str. 3	
Do, 11.02.2010	Döbereiner Hörsaal	Analytical Ultracentrifugation II:
14.15-15.45		Interactions

Lecture 5 Analytical Ultracentrifugation II: interactions



Steve Harding





Free solution, <u>no</u> immobilisation, columns, membranes etc. needed. Self-association and hetero-interactions, stoichiometry, reversibility and strength (K_{ea} , K_{d} or ΔG^{o})

- Use of sedimentation equilibrium and sedimentation velocity
- Self association or "A-A" interactions, heterologous or "A-B" interactions
- How thermodynamic non-ideality problems are now dealt with experimentally & computationally (COVOL)
- Polysaccharide interactions, complex formation and mucoadhesion

Self-association: $A + A \leftrightarrow AA$

Association constant $K_{eq} = [AA]/[A]^2 - ml/mol, \mu M^{-1}, M^{-1}$

Dissociation constant $K_d = [A]^2/[AA] - mol/ml, \mu M, M$

Heterologous association: $A + B \leftrightarrow AB$

Association constant $K_{eq} = [AB]/[A][B] - ml/mol, \mu M^{-1}, M^{-1}$

Dissociation constant $K_d = [A][B]/[AB] - mol/ml, \mu M, M$

 $\Delta G^{o} = RTIn K_{eq} - J/mol$ K_{eq} also given the symbol K_{2} or K_{a} Interaction strengths commonly represented by K_d values

Strong interactions: $K_d < 1\mu M$

Medium interactions: $1\mu M < K_d < 20\mu M$

Weak interactions: $K_d > 20\mu M$

Very weak interactions: $K_d > 100 \mu M$

NB this applies to <u>reversible</u> interactions





monomer-dimer equilibrium. The apparent weight-average molecular masses, $M_{\rm w,app}$, obtained for interference optics and absorption optics measurements were 21 400 ± 3000 Da and 21 800 ± 4000 Da, respectively.

From these data it is possible to estimate the molar dissociation constant, K_d . For a dimerizing system, correct to first-order in concentration, c (g·mL⁻¹) [58]

$$\frac{1}{M_{\rm w,app}} = \frac{1}{M_1} + 2\left(B_{11} - \frac{K_2}{M_1^2}\right)c\tag{1}$$

 K_2 is the dimerization constant (mL·mol⁻¹), M_1 is the monomer molecular mass and B_{11} is the monomer–monomer second thermodynamic virial coefficient (in mL·mol·g⁻²). If the system is assumed to be ideal, $B_{11} \approx 0$, then K_2 is simply

$$\frac{M_1^2}{2c} \left(\frac{1}{M_1} - \frac{1}{M_{\rm w,app}} \right) \tag{2}$$

Taking $M_1 = 16\ 626\ \text{Da}$, $M_{\text{w,app}} = 21\ 600\ \pm\ 3000\ \text{Da}$ and $c = 60\ \mu\text{M}$ which is equivalent to 1 mg·mol⁻¹, K_2 is calculated to be $1.92 \times 10^6\ \text{mL·mol}^{-1}$, and K_d (equivalent to $1/K_2$) is

Sedimentation Equilibrium



action parameters, K_d etc.

Carbohydrate induced dimerisation of a protein $M \sim 17.4$ kDa, $K_d \sim (120)\mu M$



James Flint¹, Didier Nurizzo², Stephen E. Harding³, Emma Longman³ Gideon J. Davies², Harry J. Gilbert¹ and David N. Bolam^{1*}

MSTAR analysis

Sedimentation equilibrium - Electron Transfer Flavoproteinheterodimer, $M \sim 63$ kDa, $K_d \sim (1.5\pm0.1)\mu M$



Helmut Cölfen · Stephen E. Harding Emma K. Wilson · Nigel S. Scrutton Donald J. Winzor

European Biophys. J. (1997)

Low temperature solution behaviour of *Methylophilus methylotrophus* electron transferring flavoprotein: a study by analytical ultracentrifugation





2.M_{w,app} (r) analysis - reversibility



3. Fitting the concentration distribution – K_d

Sedimentation equilibrium - Electron Transfer Flavoproteinheterodimer, $M \sim 63$ kDa, $K_d \sim (1.5\pm0.1)\mu M$



1. M_{w.app} analysis vs loading conc - stoichiometry.



Blue chain: 29kDa; Purple chain: 34kDa



2.M_{w,app} (r) analysis - reversibility



3. Fitting the concentration distribution – K_d



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Sedimentation Equilibrium



Choice: 1.Ignore (valid only for strong interactions) 2.Calculate it on the basis of excluded volume theory



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Excluded Volume for Pairs of Triaxial Ellipsoids at Dominant Brownian Motion

J. M. RALLISON* AND S. E. HARDING†

*University of Cambridge, Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW, and †University of Cambridge, Department of Biochemistry, Tennis Court Road, Cambridge CB2 1QW, England

Received May 4, 1984; accepted July 19, 1984

An expression is given for the volume excluded by one rigid convex particle to another when Brownian motion dominates the orientation statistics. Explicit numerical results are presented for pairs of triaxial ellipsoids. Implications for the determination of macromolecular size and shape by measurement of the second virial coefficient are discussed. © 1985 Academic Press, Inc. $u_{\rm AB} = V_{\rm A} + V_{\rm B} + S_{\rm A}R_{\rm B} + S_{\rm B}R_{\rm A}$

$$S = \frac{8}{5} \int_{0}^{\pi/2} \int_{0}^{\pi/2} \cos u du dv \left\{ \left(\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \right) \Delta - \sin^{2} v \cos^{2} v \cos^{2} u \Delta^{3} c \left(\frac{b}{a} - \frac{a}{b} \right) \left(\frac{1}{a^{2}} - \frac{1}{b^{2}} \right) - \sin^{2} u \cos^{2} u \Delta^{3} \left(\frac{\cos^{2} v}{a^{2}} + \frac{\sin^{2} v}{b^{2}} - \frac{1}{c^{2}} \right) \left[c \left(\frac{b \cos^{2} v}{a} + \frac{a \sin^{2} v}{b} \right) - \frac{ab}{c} \right] \right\}$$
[3]
and
$$R = \frac{2}{3\pi} \int_{0}^{\pi/2} \int_{0}^{\pi/2} \cos u du dv \left\{ \left(\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab} \right) \Delta^{2} - \sin^{2} v \cos^{2} v \cos^{2} u \Delta^{4} \left(\frac{1}{a^{2}} - \frac{1}{b^{2}} \right) \frac{1}{c} \left(\frac{b}{a} - \frac{a}{b} \right) - \sin^{2} u \cos^{2} u \Delta^{4} \left(\frac{\cos^{2} v}{a^{2}} + \frac{\sin^{2} v}{b^{2}} - \frac{1}{c^{2}} \right) \left[\frac{1}{c} \left(\frac{b \cos^{2} v}{a} + \frac{a \sin^{2} v}{b} \right) + \frac{c}{ab} - \frac{b}{ac} - \frac{a}{bc} \right] \right\}.$$
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theory quite complicated! ... but built into a simple to use program called COVOL

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[4]

COVOL – calculates B from size, shape and charge

CD2-CD48 cell recognition heterodimer: a weak interaction





2BM (from COVOL programme) = 10.4 ml/g

K_d = 100<u>+</u>30 μM

 K_d (from SPR) = 70-90 μ M (Anton van der Merwe)

Helena Silkowski · Simon J. Davis · A. Neil Barclay Arthur J. Rowe · Stephen E. Harding · Olwyn Byron European Biophys. J. (1997)

Characterisation of the low affinity interaction between rat cell adhesion molecules CD2 and CD48 by analytical ultracentrifugation





Biochemistry (1998)

Structure and Mucoadhesion of Mussel Glue Protein in Dilute Solution[†]



Biochemistry (1998)











- numerical solutions to Lamm equation (1923)



- nb single peak does not necessarily mean single solute! (Gilbert & Jenkins 1958)



Sedimentation Velocity



<u>Shape</u>, molecular weight & associated mass-action parameters, K_d etc.

<u>Choice of software</u>: 1.SEDFIT (P. Schuck): analysis of c(r,t) in terms of stoichiometry, reversibility

2.SC-ISOTHERM (A. Rowe et al) analysis of weight average sed.coeff. vs conc. in terms of stoichiometry & K_d

3.SEDANAL (W. Stafford): global analysis of several c(r,t)'s in terms of K_d , k_{off} (& k_{on})



of Maryland, College Park, MD 20742





...as concentration increases, proportion of dimers increases

J Mol Biol. 2008 June 27; 380(1): 223–236. doi:10.1016/j.jmb.2008.04.068.

Kinetic Partitioning Between Alternative Protein: Protein

Interactions Controls a Transcriptional Switch

Huaying Zhao and Dorothy Beckett^{*} Department of Chemistry & Biochemistry, Center for Biological Structure & Organization, University of Maryland, College Park, MD 20742



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anything faster than 0.01 per second is considered as instantaneous. Too slow – anything slower than 0.0001 per second won't distort the boundary enough







J Mol Biol. 2008 June 27; 380(1): 223-236. doi:10.1016/j.jmb.2008.04.068.

Kinetic Partitioning Between Alternative Protein: Protein

Interactions Controls a Transcriptional Switch

Huaying Zhao and Dorothy Beckett^{*} Department of Chemistry & Biochemistry, Center for Biological Structure & Organization, University of Maryland, College Park, MD 20742 K_d (hetero-dimerisation) ~(2.4<u>+</u>0.4) μM

SC-ISOTHERM analysis CD2:CD48





Fig. 6 Weight-average sedimentation coefficients (\bullet) modelled iteratively to Eqs. (7)–(11) [with k_s (monomer) set as 5 ml/g; k_s (dimer as 8.5 ml/g)], for values of the dissociation constant K_d in the range 8–120 µM using the software *SA-Plot*

Helena Silkowski · Simon J. Davis · A. Neil Barclay Arthur J. Rowe · Stephen E. Harding · Olwyn Byron

Characterisation of the low affinity interaction between rat cell adhesion molecules CD2 and CD48 by analytical ultracentrifugation Conformation analysis in <u>associating</u> systems

e.g. take the case of a dimerising globular protein system



Neurophysin dimerises side by side - triaxial contour mapping (ELLIPS3)



Neurophysin dimerises side by side - triaxial contour mapping (ELLIPS3)



and is bioprocessing-induced aggregation in antibodies linked to a conformation change in the monomer?...



Polysaccharide interactions

- Mucoadhesive interactions chitosan
- Weak dimerisation arabinoxylan
- Self association of amino-celluloses

Mucoadhesive interactions - chitosan

1. Image by atomic force microscopy



2. Sedimentation velocity result

Sedimentation coefficient $s_{20,w}^{o} \sim 1S$

Deacon et al, Biochem. J. 2000

Chitosan – mucin complex

1. Image by atomic force microscopy



2. Sedimentation velocity result

Sedimentation coefficient $s^{o}_{20,w} \sim 2000S$

Deacon et al, Biochem. J. 2000

Chitosan – mucin complex

1. Image by atomic force microscopy



2. Sedimentation velocity result

Sedimentation coefficient s°_{20,w} ~ 2000S

very strong, irreversible interaction

A very weak carbohydrate interaction: arabinoxylan dimerisation



very weak, reversible interaction

A very weak carbohydrate interaction: arabinoxylan dimerisation



very weak, reversible interaction

A very weak carbohydrate interaction: arabinoxylan dimerisation



very weak, reversible interaction

Patel et al, Biophys. J. 2007

Protein-like self-association in amino-celluloses!



Daus et al, (mss in preparation)

s ~ M^b



Summary

- free solution technique for K_d's from <1μM to >100μM, and irreversible aggregation (e.g. antibodies) and complex formation phenomena (e.g. mucoadhesive complexes)
- major advances in both sedimentation velocity and equilibrium for stoichiometry, reversibility and strengths, self and hetero-associations and conformation analysis
- for reversible associations, complications through nonideality, often need to be considered – COVOL, for prediction of B.

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